Math 33A :

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(Zoom link on syllabus & website)

Functions :

Notation : $\mathbb{R}$: the set of all real numbers

$$\Big\{ \underset{\text{function}}{f} : \underset{\text{domain}}{\mathbb{R}} \longrightarrow \underset{\text{codomain}}{\mathbb{R}} \Big\}$$

For every real number x, we have another real number $f(x)$

Ex : $f(x) = x$, $f(x) = x^2$, $f(x) = 0$, $f(x) = \sin(x)$

Not^n : $\mathbb{R}^2$: set of all pairs of real numbers

$$\text{components} \rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Big\} \text{ vector}$$

ex : $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} \pi \\ -1000 \end{bmatrix}$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \end{bmatrix}$$

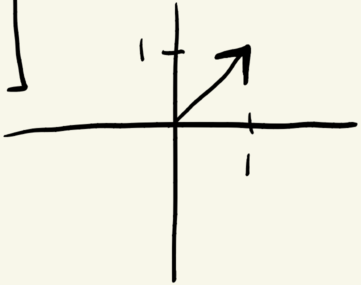
<u>add</u>

c real number

$$\underset{\substack{\uparrow \\ \text{scalar}}}{c} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c x_1 \\ c x_2 \end{bmatrix}$$

<u>scale</u>

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



$\mathbb{R}^3$, set of all 3-component vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\mathbb{R}^4$ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, ..., $\mathbb{R}^n$: set of all vectors w/ n components

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$f : \underline{\mathbb{R}^2} \longrightarrow \underline{\mathbb{R}}$$

$$f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = x_1 + x_2$$

$$g : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$$

$$g\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_3 \end{bmatrix}$$

Systems of Linear Equations $\xleftrightarrow{\text{Matrices}}$ Linear Transformations $\Big\}$ special type of function w/ nice geometric properties

Function Composition:

$$f(x) = x^2, \quad g(x) = e^x$$

$$(g \circ f)(x) = g(f(x)) = g(x^2) = e^{x^2}$$

$$T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3, \quad S : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$$

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_1 \\ x_2 \end{bmatrix} \qquad S\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} y_1 + y_2 \\ y_3 + y_2 \end{bmatrix}$$

$$(S \circ T)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = S\left(\begin{bmatrix} x_1 \\ x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_1 \\ x_2 + x_1 \end{bmatrix}$$

$T: \mathbb{R} \longrightarrow \enclose{circle}{\mathbb{R}^3}$, $\quad S: \enclose{circle}{\mathbb{R}^2} \longrightarrow \mathbb{R}^2$

Inverses:

$I_n: \mathbb{R}^n \longrightarrow \mathbb{R}^n$ identity "do-nothing"

$$I_n\left(\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$f: \mathbb{R}^n \longrightarrow \mathbb{R}^m$,

$f^{-1}: \mathbb{R}^m \longrightarrow \mathbb{R}^n$ is the inverse of f

if $\quad f \circ f^{-1} = I_m$, $\quad f^{-1} \circ f = I_n$

ex: $f(x) = \sqrt[3]{x}$, $\quad f^{-1}(x) = x^3 \qquad f(f^{-1}(x)) = f(x^2) = \sqrt[3]{x^3} = x$

$$f^{-1}(f(x)) = f^{-1}(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$$

ex: $f(x) = 0$